LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2019**

16/17/18UMT1MC01– ALGEBRA AND CALCULUS - I

Date: 03-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART-A

ANSWER ALL QUESTIONS. **(2x10=20)**

1. Find the n^{th} derivative of e^{ax}.
2. Find the slope of the tangent to the curve $r = a(1 - \cos\theta)$at $= \frac{\pi}{2}$.
3. Write the necessary condition for maxima and minima of the function $f(x, y)$.
4. Define Saddle point.
5. Find the radius of curvature at the point (x, y) on the curve $y = c \cosh\left(\frac{x}{c}\right)$.
6. Find the pedal equation of a curve $r = ae^{\theta cot\alpha}$.
7. Define Reciprocal equation.
8. Form the equation one of whose root is $2 - \sqrt{-3}$.
9. State Descarte's rule of sign.
10. Increase by 2 the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

PART-B

Answer any FIVE questions. **(5x8=40)**

11. Show that the parabolas $r = a\, sec^2\frac{\theta}{2}$ and $r = b\, cosec^2\frac{\theta}{2}$ intersect at right angle.
12. Find the minimum value of the function $4x^2 + 9y^2 + 6xy - 8x - 24y + 4$.
13. Find the radius of curvature at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ to the curve $\sqrt{x} + \sqrt{y} = 1$.
14. Solve the equation $3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 = 0$.Given that $\sqrt{2} + \sqrt{5}$ is a root of it.
15. Show that the equation $3x^5 - 2x^3 - 4x + 2 = 0$ has three real and two imaginary roots.
16. Find the radius of curvature at any point on the curve $r^n = a^n \cos n\theta$.
17. Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$, given that two of its roots are equal.
18. Find the n^{th} derivative of $\sin 2x \sin 4x \sin 6x$.

PART-C

Answer any TWO questions. **(2x20=40)**

19. (a) If $y = a\cos(\log x) + b\sin(\log x)$. Prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

    (b) Find the minimum value of $x^2 + y^2 + z^2$ subject to the constraint $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

(10+10)

20. Find the Evolute of the Rectangular Hyperbola $xy = c^2$.

21. (a) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in Arithmetic Progression.

    (b) Solve : $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$. **(05+15)**

22. Using Horner's method, find the positive root of $x^3 - 3x + 1$ which lies between 1and 2 , correct to two decimal places.

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